

Renormalization group analysis of cosmological constraint on the mass of Higgs scalar

V. V. Kiselev^{1),2)}, S. A. Timofeev^{1),2)*}

¹⁾*Institute for High Energy Physics, Protvino, Russia*

²⁾*Moscow Institute of Physics and Technology, Dolgoprudny, Russia*

The Higgs boson of Standard Model, minimally coupled to the gravitation, is not able to produce the inflation of early Universe if its mass exceeds the threshold value, which is equal to $m_H^{\min} = 142$ GeV in the tree approximation for the scalar potential. Two-loop corrections modify the estimate as $m_H^{\min} = 150 \pm 3$ GeV, so that higher-order corrections of perturbation theory are completely under control, though they are numerically important in respect of experimental searches.

I. INTRODUCTION

The stage of inflationary expansion of Universe was recently accepted as the reference model for the evolution of early Universe [1–5]. In [6] we have considered the properties of inflation, produced by the scalar Higgs boson of Standard model with the minimal coupling to the gravitation, i.e., when the term of lagragian in the form of $\xi \Phi^\dagger \Phi R$ gets zero constant $\xi = 0$ (here Φ is the Higgs field, R is the scalar curvature). Since the vacuum expectation value of Higgs field is negligible with respect to the Planck scale of energy, characteristic for the inflation regime, one can neglect terms quadratic to the field in potential $V(\Phi)$, so that the potential is reduced to the form of $V = \lambda(\Phi^\dagger \Phi)^2$. The inflation with the quartic self-action of inflaton was in detail studied in the framework of slowly drifting stable attractor appearing in the system of field equations to the leading approximation of flat homogeneous and isotropic Universe [7, 8]. In [6] we have shown that exclusion of inflation produced by the Higgs boson results in the constraint on the constant λ : $\lambda > \frac{1}{6}$, that leads to a critical minimal value of boson mass. This constraint is related with the fact that the inflation produced by the Higgs field is finished at the Hubble constant H determining the rate of Universe expansion as given by the formula [6]

$$2\pi G H^2 = \lambda, \quad (1)$$

where G is the gravitational constant. The inflation cannot be produced by the Higgs field, if the constant of self-action is close to unit, hence, the Hubble constant to the end of inflation would be of the order of Planck energy. But the Planck scales of energy density cannot be described in the framework of classical theory of gravitation, which is the necessary ingredient of inflationary model. The numerical consideration of scheme for the mentioned derivation of decoupling constant of self-action for the Higgs scalar gives the critical value of $\lambda_c = \frac{1}{6}$ by defining a limit of quantum gravity in cosmology. Then, the mass of Higgs field should exceed the decoupling value¹ equal to $m_H^{\min} = 142$ GeV to the tree approximation for the potential. Then, after the determination of decoupling mass to the leading order, one has got the problem to take into account loop corrections. These corrections depend on the energy scale, corresponding to the phenomenon under consideration, therefore we use the renormalization group up to two loops in order to account for the higher corrections of perturbation theory. First, we estimate the energy scale relevant to the end of inflation. Second, we study the dependence of final result on the initial data and scale variations fixing the running constants and masses.

II. ESTIMATING THE ENERGY SCALE

We can estimate the characteristic scale of energy in the threshold region of inflationary regime in several methods.

*E-mail address: serg.timofeev@mail.ru

¹ We suggest here that the Higgs boson is the only scalar field in the theory, so that the Higgsian inflation is not consistent with observations, while the only possible variant is the ordinary Big Bang due to oscillations of Higgs field in vicinity of minimum of its potential with a fine tuning of initial data consistent with the large scale structure of Universe. The case with introducing the additional scalar field responsible of the inflation and two field dynamics in early Universe will be considered elsewhere.

A. Characteristic value of field

Let the running constant of self-action be given at the scale fixed by the field value $\lambda = \lambda(\phi)$, where the real electrically neutral field ϕ is given by the gauge $\phi = \Phi\sqrt{2}$. Then, we set on $\lambda = \frac{1}{6}$ in the relation for the threshold value of Hubble constant in (1), and we make use of Einstein equations taking into account the fact that at the threshold the kinetic energy is twice the potential one [6], hence,

$$H^2 = 2\pi G\lambda\phi^4, \quad (2)$$

we find at $\mu = \phi$:

$$\mu = \sqrt{\frac{1}{2\pi G}} \approx 4.9 \times 10^{18} \text{ GeV}. \quad (3)$$

B. The energy density

The scale of energy can be estimated by the value of energy density by $\rho = \mu^4$, so that due to the Einstein equations

$$H^2 = \frac{8\pi G}{3}\rho, \quad (4)$$

we get

$$\mu = \sqrt{\frac{1}{4\pi G\sqrt{2}}} \approx 2.9 \times 10^{18} \text{ GeV}, \quad (5)$$

that very slightly differs from the result in (3).

C. The field virtuality

The displacement of virtual field from the mass shell gives $\mu^2 = m^2 - p^2$, where the mass equals to

$$m^2 = \frac{\partial^2 V}{\partial \phi^2} = 3\lambda\phi^2 = \frac{1}{4\pi G},$$

while the 4-momentum p is estimated by

$$p_0\phi = i\frac{\partial \phi}{\partial t},$$

whereas $\mathbf{p} = 0$, since the Higgs field is spatially homogeneous and isotropic in the reference system under consideration. Finally, we get

$$p_0^2 = -\frac{(\dot{\phi})^2}{\phi^2} = -\frac{1}{12\pi G}.$$

Here we have used the values of ϕ and $\dot{\phi}$, as derived in [6]. Thus, we find

$$\mu^2 = m^2 - p^2 = \frac{1}{3\pi G}, \quad (6)$$

$$\mu = \sqrt{\frac{1}{3\pi G}} \approx 4.0 \times 10^{18} \text{ GeV}. \quad (7)$$

Therefore, we conclude that the energy scale has got the Planckian order, so that it is estimated as $3 \times 10^{18} \text{ GeV}$.

III. THE RENORMALIZATION GROUP ANALYSIS

According to [9], the experimental values of masses and coupling constants entering as initial data into the renormalization group equations are equal to

$$\begin{aligned} m_Z &= 91.1873 \pm 0.0021 \text{ GeV}, \\ m_t &= 170.9 \pm 1.9 \text{ GeV}, \\ \alpha_{\text{em}}^{-1}(m_Z) &= 127.906 \pm 0.019, \\ \alpha_s(m_Z) &= 0.1187 \pm 0.0020, \\ \sin^2 \theta_W &= 0.2312 \pm 0.002, \end{aligned} \tag{8}$$

wherein the running constants are normalized at the scale equal to the mass of Z -boson, while the sine of Weinberg angle is standardly defined in terms of renormalized values of coupling constants in the electroweak group, g and g' .

It is convenient to take those quantities at scale $\mu = m_t$, so that

$$\begin{aligned} g' &= 0.358765 \pm 0.00010, \\ g &= 0.648532 \pm 0.00039, \\ g_s &= 1.17372 \pm 0.0099. \end{aligned} \tag{9}$$

The relation between the running constant $\lambda(\mu)$ and Yukawa constant $h_t(\mu)$ for the t -quark at scale $\mu = m_t$ is given in terms of Higgs scalar mass and t -quark mass, m_t , as

$$\lambda(m_t) = \frac{m_H^2}{2v^2}(1 + \Delta_H), \tag{10}$$

$$h_t(m_t) = \frac{\sqrt{2}}{v}m_t(1 + \Delta_t), \tag{11}$$

where the vacuum expectation value of Higgs field $v = 246.2 \text{ GeV}$ is related to the Fermi constant of weak interaction, while corrections $\Delta_{H,t}$ are given in Appendix 1 up to the 1-loop accuracy (formulas of calculations refer to the scheme $\overline{\text{MS}}$ and they are taken from [10].).

The renormalization group equations (see Appendix 2 extracted from [10]) show that the critical value of Higgs boson mass is displaced from its tree level value by 10 GeV upper in the 1-loop approximation and by 8 GeV upper in 2 loops.

We calculate the dependence of result on the initial data such as α_s , m_t and μ , since they involve the main uncertainty into the estimates of cosmological constraint on the Higgs boson mass, as we find numerically.

Then, in 2-loop approximation the dependence of critical value of Higgs scalar mass m_H^{min} on the parameters can be presented in terms of partial derivatives, so that

$$\frac{\partial m_H^{\text{min}}}{\partial \ln \mu} = -0.28 \text{ GeV}, \tag{12}$$

$$\frac{\partial m_H^{\text{min}}}{\partial \alpha_s(m_t)} = -110 \text{ GeV}, \tag{13}$$

$$\frac{\partial m_H^{\text{min}}}{\partial m_t} = 1.0. \tag{14}$$

We transform the derivative with respect to the strong interaction constant α_s from scale m_t in (13) to scale m_Z . So, we have used formulas from Appendix 2 for the loop corrections to charges. In 1-loop approximation we find

$$\frac{1}{\alpha_s(m_Z)} = \frac{1}{\alpha_s(m_t)} + \frac{\beta_0}{2\pi} \ln \frac{m_Z}{m_t}, \tag{15}$$

where $\beta_0 = 11 - \frac{2}{3}n_f$, and n_f is the number of active quark flavors at scales $m_Z < \mu < m_t$, i.e. $n_f = 5$. Thus,

$$\frac{\partial m_H^{\text{min}}}{\partial \alpha_s(m_Z)} = -93.8 \text{ GeV}. \tag{16}$$

Therefore, the results can be presented in the form

$$m_H^{\min} = 150 + 0.28 \ln \frac{10^{18}}{\mu} - 0.19 \frac{\alpha_s - 0.1187}{0.002} + 2 \frac{m_t - 171}{2} \pm 2 \text{ GeV}, \quad (17)$$

where μ and m_t are expressed in GeV.

We see from (17) that the dependence on the energy scale is weak, therefore, the order of magnitude for the scale is important, only, as we have expected above. The strong dependence on the mass of t -quark is evident. Further, the difference between the 1- and 2-loop results is about 2 GeV, so we can conservatively prescribe its value to the uncertainty due to higher orders of perturbation theory in the framework of renormalization group as shown in (17).

IV. DISCUSSION AND CONCLUSION

We have calculated the critical value of Higgs boson mass m_H^{\min} in the Standard model minimally coupled to the gravitation up to 2-loop corrections in the effective action. The critical mass determines the cosmological constrain. In the framework of renormalization group we have estimated variations of m_H^{\min} with respect to small changes of initial data related to the uncertainties of experimental measurements as well as at different prescriptions for the energy scale characteristic for the final stage of Universe inflation, which can be produced by the Higgs field. The obtained result is rather stable with respect to higher corrections of perturbation theory, so that the uncertainty of calculations due to this factor gives the value of 2 GeV. The other significant source of uncertainty of calculations is the mass of t -quark. Finally, we deduce the value of decoupling mass from cosmology as $m_H^{\min} = 150 \pm 3 \text{ GeV}$. In the Standard model there is the upper limit on the Higgs boson mass $m_H < 189 \text{ GeV}$, appearing from the following requirement: the Landau pole in the constant of self-action for the field should be posed at the scale higher than the Planck mass. Some cosmological consequences because of the Higgs scalar participation in the inflation, were considered in [10] with account of thermal and quantum fluctuations of Higgs field.

Our result should be compared with the model, wherein the Higgs boson of Standard model is coupled to the gravitation due to the nonminimal interaction with the constant of $\xi \sim 10^4$ [11–20]. In that case a conformal transformation allows one to express the Higgs scalar in terms of new effective scalar field minimally coupled to the gravitation, so that an effective potential includes a plateau with a scale of energy, which is $\sqrt{\xi}$ times less than the Planck mass. Then the inflation becomes admissible due to the effective field, which parameters are in a consistent agreement with observed data in cosmology, if the mass of such the Higgs scalar is constrained within the interval $135.6 \text{ GeV} < m_H < 184.5 \text{ GeV}$ (see details in [20]). The bound on the Higgs boson mass derived in our paper in the framework of minimal coupling to the gravitation, is greater than the lower limit for the case of nonminimal interaction, while the upper limits are similar in both cases.

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Appendix 1

Corrections to the constant of Higgs self-action and Yukawa constant are given by

$$\begin{aligned} \Delta_t = & -\frac{4\alpha_s(m_t)}{3\pi} + (1.0414N_f - 14.3323) \left[\frac{\alpha_s(m_t)}{\pi} \right]^2 - \frac{4\alpha(m_t)}{9\pi} + \\ & + \frac{h_t^2}{32\pi^2} \left[\frac{11}{2} - r + 2r(2r - 3) \ln(4r) - 8r^2 \left(\frac{1}{r} - 1 \right)^{3/2} \arccos \sqrt{r} \right] - \\ & - 6.90 \times 10^{-3} + 1.73 \times 10^{-3} \ln \frac{m_H}{300 \text{ GeV}} - 5.82 \times 10^{-3} \ln \frac{m_t}{175 \text{ GeV}}, \quad (\text{A.1}) \end{aligned}$$

where $r = \frac{m_H^2}{4m_t^2}$, $\alpha_s = g_s^2/(4\pi)$, and $N_f = 5$,

$$\Delta_H = \frac{G_F}{\sqrt{2}} \frac{m_z^2}{16\pi^2} [\xi f_1(\xi) + f_0(\xi) + \xi^{-1} f_{-1}(\xi)], \quad (\text{A.2})$$

where

$$f_1(\xi) = 6 \ln \frac{m_t^2}{m_H^2} + \frac{3}{2} \ln \xi - \frac{1}{2} Z \left(\frac{1}{\xi} \right) - Z \left(\frac{c_w^2}{\xi} \right) - \ln c_w^2 + \frac{9}{2} \left(\frac{25}{9} - \frac{\pi}{\sqrt{3}} \right), \quad (\text{A.3})$$

$$f_0(\xi) = -6 \ln \frac{m_t^2}{m_z^2} \left[1 + 2c_w^2 - 2\frac{m_t^2}{m_z^2} \right] + \frac{3c_w^2 \xi}{\xi - c_w^2} \ln \frac{\xi}{c_w^2} + 2Z \left(\frac{1}{\xi} \right) + 4c_w^2 Z \left(\frac{c_w^2}{\xi} \right) + \left(\frac{3c_w^2}{s_w^2} + 12c_w^2 \right) \ln c_w^2 - \frac{15}{2} (1 + 2c_w^2) - 3\frac{m_t^2}{m_z^2} \left(2Z \left(\frac{m_t^2}{m_z^2 \xi} \right) + 4 \ln \frac{m_t^2}{m_z^2} - 5 \right), \quad (\text{A.4})$$

$$f_{-1}(\xi) = 6 \ln \frac{m_t^2}{m_z^2} \left[1 + 2c_w^4 - 4\frac{m_t^4}{m_z^4} \right] - 6Z \left(\frac{1}{\xi} \right) - 12c_w^4 Z \left(\frac{c_w^2}{\xi} \right) - 12c_w^4 \ln c_w^2 + 8(1 + 2c_w^4) + 24\frac{m_t^4}{m_z^4} \left[\ln \frac{m_t^2}{m_z^2} - 2 + Z \left(\frac{m_t^2}{m_z^2 \xi} \right) \right], \quad (\text{A.5})$$

with notations: $\xi = \frac{m_H^2}{m_z^2}$, $s_w^2 = \sin^2 \theta_W$, $c_w^2 = \cos^2 \theta_W$, where θ_W is the Weinberg angle,

$$Z(z) = \begin{cases} 2A \arctan(1/A) & (z > 1/4), \\ A \ln[(1+A)/(1-A)] & (z < 1/4), \end{cases} \quad A = \sqrt{|1-4z|}.$$

Appendix 2

The 2-loop equations of renormalization group (RG) for charges take the following form:

$$\frac{dg_i}{dt} = \kappa g_i^3 b_i + \kappa^2 g_i^3 \left(\sum_{j=1}^3 B_{ij} g_j^2 - d_i^t h_t^2 \right), \quad (\text{A.6})$$

where $t = \ln \mu$, and $\kappa = 1/(16\pi^2)$, while

$$b = (41/6, -19/6, -7), \quad B = \begin{pmatrix} 199/18 & 9/2 & 44/3 \\ 3/2 & 35/6 & 12 \\ 11/6 & 9/2 & -26 \end{pmatrix}, \quad d^t = (17/6, 3/2, 2). \quad (\text{A.7})$$

The RG equation for the Yukawa constant is written down as

$$\frac{dh_t}{dt} = \kappa h_t \left(\frac{9}{2} h_t^2 - \sum_{i=1}^3 c_i^t g_i^2 \right) + \kappa^2 h_t \left(\sum_{ij} D_{ij} g_i^2 g_j^2 + \sum_i E_i g_i^2 h_t^2 + 6(\lambda^2 - 2h_t^4 - 2\lambda h_t^2) \right), \quad (\text{A.8})$$

where

$$c^t = (17/12, 9/4, 8), \quad D = \begin{pmatrix} 1187/216 & 0 & 0 \\ -3/4 & -23/4 & 0 \\ 19/9 & 9 & -108 \end{pmatrix}, \quad E = (131/16, 225/16, 36). \quad (\text{A.9})$$

The RG equation for the constant of self-action of Higgs boson reads off

$$\begin{aligned} \frac{d\lambda}{dt} = \kappa \Big\{ & -6h_t^4 + 12h_t^2 \lambda + \frac{3}{8} [2g_4 + (g^2 + g'^2)^2] - 3\lambda(3g^2 + g'^2) + 24\lambda^2 \Big\} + \\ & + \kappa^2 \Big\{ 30h_t^6 - h_t^4 \left(32g_s^2 + \frac{8}{3}g'^2 + 3\lambda \right) + h_t^2 \left[-\frac{9}{4}g^4 + \frac{21}{2}g^2 g'^2 - \frac{19}{4}g'^4 + \right. \\ & + \lambda \left(80g_s^2 + \frac{45}{2}g^2 + \frac{85}{6}g'^2 - 144\lambda \right) \Big] + \frac{1}{48} (915g^6 - 289g^4 g'^2 - 559g^2 g'^4 - 379g'^6) + \\ & \left. + \lambda \left(-\frac{73}{8}g^4 + \frac{39}{4}g^2 g'^4 + \frac{629}{24}g'^4 + 108\lambda g^2 + 36\lambda g'^2 - 312\lambda^2 \right) \right\}. \quad (\text{A.10}) \end{aligned}$$

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